

# Nuclear Structure Aspects of Neutrinoless Double Beta Decay

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We decompose the neutrinoless double-beta decay matrix elements into sums of products over the intermediate nucleus with two less nucleons. We find that the sum is dominated by the  $J^\pi = 0^+$  ground state of this intermediate nucleus for both the light and heavy neutrino decay processes. This provides a new theoretical tool for comparing and improving nuclear structure models. It also provides the connection to two-nucleon transfer experiments.

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Neutrinoless double beta decay  $0\nu\beta\beta$  is one of the most important current topics in physics that provides unique information on the neutrino properties [1], [2], [3]. The  $0\nu\beta\beta$  decay process and the associated nuclear matrix elements (NME) were investigated by using several approaches including the quasiparticle random phase approximation (QRPA) [1], the interacting shell model [4], [5], the interacting boson model [6], [7], the generator coordinate method [8], and the projected Hartree-Fock Bogoliubov model [9]. It is critical to assess which nuclei are the best candidates for experimental study.

Since the experimental decay rate is proportional to the square of the calculated nuclear matrix elements, it is important to calculate these matrix elements with high accuracy to be able to extract the neutrino effective mass which can be used to determine the absolute scale of neutrino masses. However, the theoretical methods used give results that differ from one another by factors of up to 2-3. It is important to understand the nuclear structure aspects of these matrix elements and why the models give differing results. In this Letter we present a new theoretical tool for understanding  $0\nu\beta\beta$  matrix elements by expanding them in terms of a summation over states in the nucleus with two less nucleons ( $A - 2$ ). We show that the matrix elements are dominated by the contribution through the ground state of the ( $A - 2$ ) intermediate nucleus. We also show that the light-neutrino matrix elements are dominated by the Gamow-Teller type operator that is proportional to a schematic interaction of the form  $\sigma_1 \cdot \sigma_2/r$ . This opens up new ways of comparing theoretical models and improving the accuracy of the NME for  $0\nu\beta\beta$  decay.

The  $0\nu\beta\beta$  process can be naturally described in  $2^{nd}$  order perturbation theory, in which the energies of the virtual states of the intermediate nucleus obtained by a single beta decay of the parent nucleus enter into the propagator. However, it has been known for some time (see e.g. [10], [11] and references therein) that these energies are small compared to the neutrino exchange energy, and therefore the widely used closure approximation replaces these energies by a constant value and sums-out the contribution of the intermediate states. It was shown

[10], [12], [11] that this approximation provides matrix elements about 10% smaller, but we recently found [10], [12] optimal closure energies for which the nuclear matrix elements in both approaches are the same (see e.g. Fig. 5 of Ref. [12]). Therefore in this letter, for the light neutrino exchange matrix elements we use closure approximation with the optimal closure energies, which are 0.5 MeV, 3.5 MeV and 3.5 MeV for  $^{48}\text{Ca}$ ,  $^{76}\text{Ge}$ , and  $^{82}\text{Se}$ , respectively. The heavy neutrino exchange matrix elements [1], [5] do not depend on the energies of the intermediate states.

We will start with the case for the  $0\nu\beta\beta$  decay of  $^{76}\text{Ge}$  that is shown in Fig. 1. Previously, the structure dependence has been analyzed in terms of the “charge-exchange” to intermediate states in  $^{76}\text{As}$ . In contrast, we will show the results for expanding in terms of the intermediate states in  $^{74}\text{Ge}$  represented by the red arrow in Fig. 1 that provide a simpler understanding of the nuclear structure dependence. We will also show results for the  $0\nu\beta\beta$  decay of  $^{48}\text{Ca}$  and  $^{82}\text{Se}$ .

The results for  $^{76}\text{Ge}$  and  $^{82}\text{Se}$  were obtained in the  $jj44$  model space with the set of four orbitals ( $0f_{5/2}, 1p_{3/2}, 1p_{1/2}, 0g_{9/2}$ ) for both protons and neutrons. We use the JUN45 Hamiltonian [13] for the  $jj44$  model space. The results for  $^{48}\text{Ca}$  were obtained for the  $pf$  model space with the set of four orbitals ( $0f_{7/2}, 0f_{5/2}, 1p_{3/2}, 1p_{1/2}$ ) for both protons and neutrons. We use the GXPF1A Hamiltonian [14] for the  $pf$  model space. We use the shell-model computer code NuShellX [15].

The nuclear matrix element  $M^{0\nu}$  can be presented as a sum of Gamow-Teller ( $M_{GT}^{0\nu}$ ), Fermi ( $M_F^{0\nu}$ ), and Tensor ( $M_T^{0\nu}$ ) matrix elements (see, for example, Refs. [10], [16]),

$$M^{0\nu} = M_{GT}^{0\nu} - \left( \frac{g_V}{g_A} \right)^2 M_F^{0\nu} + M_T^{0\nu}, \quad (1)$$

where  $g_V$  and  $g_A$  are the vector and axial constants, correspondingly. In our calculations we use  $g_V = 1$  and  $g_A = 1.254$ . The  $M_\alpha^{0\nu}$  are matrix elements of scalar two-body potentials. The most important are the Gamow-

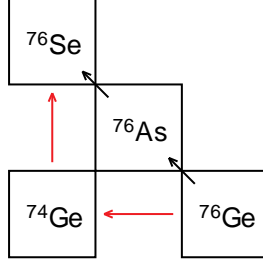


FIG. 1: Nuclei involved in the calculations for the double-beta decay of  $^{76}\text{Ge}$ .

Teller that has the form  $V_{GT}(r, A, \mu) \sigma_1 \cdot \sigma_2 \tau_1^- \tau_2^-$  and the Fermi that has the form  $V_F(r, A, \mu) \tau_1^- \tau_2^-$ , where  $\tau^-$  are the isospin lowering operators. The neutrino potentials depend on the relative distance between the two decaying nucleons,  $r$ , the mass number  $A$ , and the closure energy  $\mu$ . The radial forms are given explicitly [10]. For the heavy-neutrino exchange, the potential does not depend on  $\mu$  and it looks like a smeared-out delta function [1], [5].

The matrix element for a scalar two-body operator between an initial state  $|i\rangle = |n, \omega_i, J\rangle$  and final state  $|f\rangle = |n, \omega_f, J\rangle$  of the  $n$ -particle wave function can be expressed in the form of a product over two-body transition densities (TBTD) times two-particle matrix elements

$$\langle f | V | i \rangle = \sum_{J_o, k_\alpha \leq k_\beta, k_\gamma \leq k_\delta} \times \text{TBTD}(f, i, k, J_o) \langle k_\alpha, k_\beta, J_o | V | k_\gamma, k_\delta, J_o \rangle, \quad (2)$$

where the  $k$  stands for the set of spherical quantum numbers  $(n, \ell, j)$ . The TBTD are given by

$$\begin{aligned} \text{TBTD}(f, i, k, J_o) &= \\ &= \langle f | [A^+(k_\alpha, k_\beta, J_o) \otimes \tilde{A}(k_\gamma, k_\delta, J_o)]^{(0)} | i \rangle, \end{aligned} \quad (3)$$

where  $A^+$  is a two-particle creation operator of rank  $J_o$

$$A^+(k_\alpha, k_\beta, J_o, M_o) = \frac{[a^+(k_\alpha) \otimes a^+(k_\beta)]_{M_o}^{J_o}}{\sqrt{(1 + \delta_{k_\alpha, k_\beta})}}, \quad (4)$$

and  $\tilde{A}(k_\alpha, k_\beta, J_o) = (-1)^{J_o - M_o} A^+(k_\alpha, k_\beta, J_o, -M_o)$ . One can evaluate the TBTD by inserting a complete set of states for the  $(n-2)$  nucleon system

$$\begin{aligned} \text{TBTD}(f, i, k, J_o) &= \\ &= \sum_m \frac{\langle f | A^+(k_\alpha, k_\beta, J_o) | m \rangle \langle m | \tilde{A}(k_\gamma, k_\delta, J_o) | i \rangle}{(2J+1)} \end{aligned}$$

$$= \sum_m \text{TNA}(f, m, k_\alpha, k_\beta, J_o) \text{TNA}(i, m, k_\gamma, k_\delta, J_o), \quad (5)$$

where  $m$  stands for the quantum numbers  $(\omega_m, J_m)$  of the intermediate state with  $n-2$  nucleons.  $J_o = J_m$  when  $J = 0$ . The TNA are the two-nucleon transfer amplitudes given by

$$\text{TNA}(f, m, k_\alpha, k_\beta, J_o) = \frac{\langle f | A^+(k_\alpha, k_\beta, J_o) | m \rangle}{\sqrt{(2J+1)}}. \quad (6)$$

The TNA are normalized such that the summation over all states in the  $n-2$  nucleon system is

$$\sum_{m, k_\alpha, k_\beta} \text{TNA}(f, m, k_\alpha, k_\beta, J_o) = n(n-1)/2. \quad (7)$$

We will analyze the  $0\nu\beta\beta$  matrix elements in terms of their dependence on the intermediate states

$$\langle f | V | i \rangle = \sum_m V(f, i, m),$$

where

$$\begin{aligned} V(f, i, m) &= \sum_{k_\alpha \leq k_\beta, k_\gamma \leq k_\delta} \langle k_\alpha, k_\beta, J_o | V | k_\gamma, k_\delta, J_o \rangle \\ &\times \text{TNA}(f, m, k_\alpha, k_\beta, J_o) \text{TNA}(i, m, k_\gamma, k_\delta, J_o). \end{aligned} \quad (8)$$

The results for  $^{76}\text{Ge}$  are shown in Fig. 2 where the running sum is shown as a function of the excitation energy in  $^{74}\text{Ge}$ . The red dot shows the result obtained when all intermediate states are included as obtained from Eq. 1. We find that the NME is dominated by the contribution through the  $0^+$  ground state of  $^{74}\text{Ge}$ . This is a remarkable and simplifying result. It means that the nuclear structure aspects of this dominant term are related to the rather well studied pair transfer properties of the nuclear ground states. It is a consequence of the strong pairing interaction in the nuclear Hamiltonian. There are cancellations from intermediate states with  $J_m > 0$  up to about 6 MeV in excitation that are dominated by the  $2^+$  contributions. This cancellation reduces the total matrix element by about a factor of two for light neutrinos and about 20% for heavy neutrinos.

Fig. 3 shows the exact  $0\nu\beta\beta$  TBME for the  $jj44$  model space are compared with those of schematic interactions;  $a/r$  for light-neutrino Fermi and  $b\sigma_1 \cdot \sigma_2/r$  for light-neutrino Gamow-Teller. The exact TBME are within a few percent of those for the schematic interaction. (The heavy-neutrino TBME are closely proportional to  $a\delta(r)$  for Fermi and  $\sigma_1 \cdot \sigma_2 \delta(r)$  for Gamow-Teller). These simple schematic interactions can be used for the purpose of understanding the model dependence and nuclear structure aspects of the  $0\nu\beta\beta$  NME.

One observes that the summation over all state with GT is only about 10% smaller than the total from GT+F+T. The tensor contribution is less than 2%. The

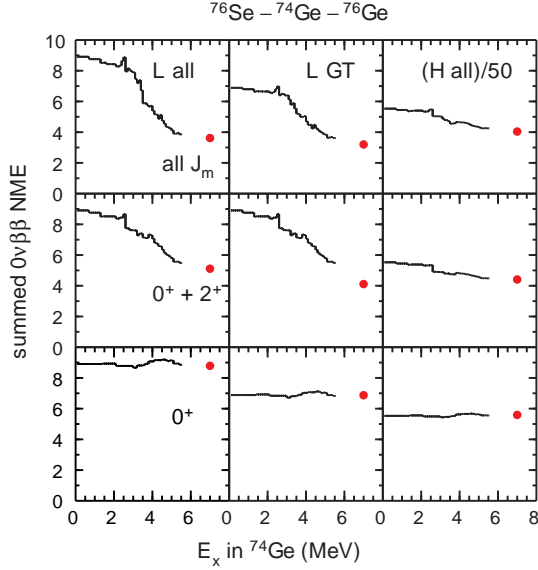


FIG. 2: (Color online) Results for  $^{76}\text{Ge}$ . The left-hand column shows the light-neutrino results (L) for the sum of the GT, F and T contributions. The middle column shows the light-neutrino results for GT contribution only. The right-hand column shows the heavy-neutrino results (H) for the sum of the GT, F and T contribution. The bottom row shows the running sums for  $0^+$  intermediate states. The middle row shows the running sums for  $0^+$  and  $2^+$  intermediate states. The top row shows the running sums for all intermediate states. The red dots are the exact results for the sum over all intermediate states.

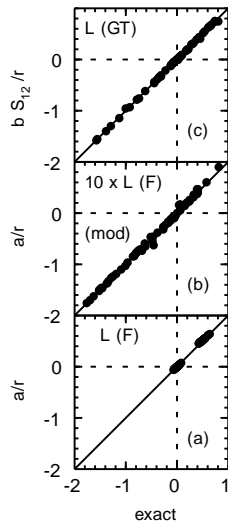


FIG. 3: Two-body matrix elements obtained from schematic interactions compared to the exact results for the  $jj44$  model space for the light neutrino (L).  $S_{12} = \sigma_1 \cdot \sigma_2$ . The modified Fermi (F) in panel (b) is that obtained from Eq. 11.

Fermi matrix elements divide into two groups in the bottom panel of Fig. 3; those near the center for the off-diagonal TBME and those near the 0.6 for the diagonal TBME. This structure is well known for the Coulomb type interaction [17]. Thus, we can write the Fermi TBME as a sum of two terms

$$\langle V_F \rangle = \langle V_{F1} \rangle + \langle V_{F2} \rangle, \quad (9)$$

where

$$\langle V_{F1} \rangle = C \delta_{k_\alpha, k_\gamma} \delta_{k_\beta, k_\delta} \quad (10)$$

and

$$\langle V_{F2} \rangle = \langle V_F \rangle - C \delta_{k_\alpha, k_\gamma} \delta_{k_\beta, k_\delta}. \quad (11)$$

For this case we take  $C = 0.6$ .  $\langle V_{F1} \rangle$  does not contribute to the  $0\nu\beta\beta$  since it conserves isospin and only goes to the IAS of the  $^{76}\text{Ge}$  ground state in  $^{76}\text{Se}$ . The second term multiplied by ten is plotted in panel (b) of Fig. 3. Thus, the effective strength of the Fermi  $0\nu\beta\beta$  operator is about a factor of ten smaller than GT and can largely be ignored for the purpose of understanding the nuclear structure aspects of the  $0\nu\beta\beta$  matrix elements.

The results for  $^{48}\text{Ca}$  and  $^{82}\text{Se}$  are shown in Figs. 4 and 5, respectively. The overall patterns are the same as seen for  $^{76}\text{Ge}$ . The results for  $^{48}\text{Ca}$  are particularly simple with 80% of the total matrix elements coming from just the  $0^+$  ground state and the first excited  $2^+$  state. We have also calculated  $^{48}\text{Ca}$  with the addition of the isospin nonconserving Hamiltonian from [17]. This allows some mixing of  $^{48}\text{Ti}$  ground state with the IAS of the  $^{48}\text{Ca}$  ground in  $^{48}\text{Ti}$ . But the mixing matrix element of 20 keV does not lead to any significant change in the result. One can also expand over intermediate states in the nucleus with two extra nucleons ( $n+2$ ), for example,  $^{78}\text{Se}$  in the case of the  $^{76}\text{Ge}$  decay. We also find that the  $J_m = 0^+$  is dominated by the ground state of the ( $n+2$ ) nucleus.

A very simple schematic diagram for the nuclear structure changes involved in double-beta decay is shown by the top row in Fig. 6. The pairing interaction enhances the two-nucleon transfers between the ground states. When one removes two neutrons one can also go to the deeper hole states shown by term (b) in Fig. 6. But adding two protons results in a state that has no overlap with the final state on the top left-hand side. However, such configurations are important because they will mix with the dominant ones in the top row due to the pairing interaction. Some of this mixing is already contained in the  $pf$  and  $jj44$  model spaces. But there will also be mixing with these configurations from outside the model space that will renormalize the  $0\nu\beta\beta$  NME.

It is well known that the two-nucleon transfer cross sections are enhanced by admixtures in wavefunction due to the pairing interaction [18], [19], and it is important to test the wavefunctions for the nuclei involved in double-beta decay by such measurements [20]. But the connection between two-nucleon transfer and the neutrino

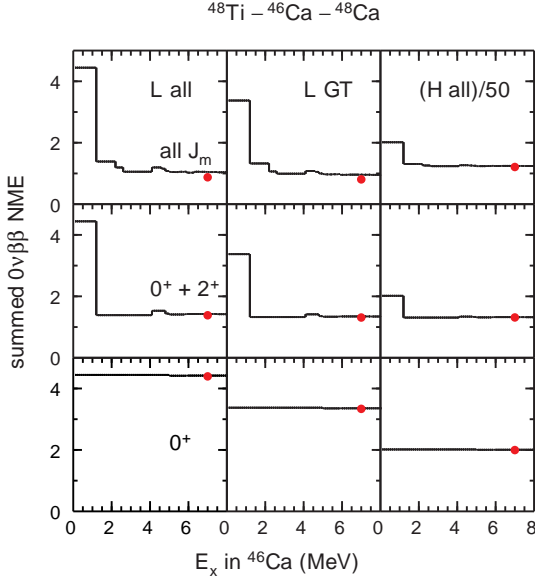


FIG. 4: (Color online) Results for  $^{48}\text{Ca}$ . See caption for Fig. 2.

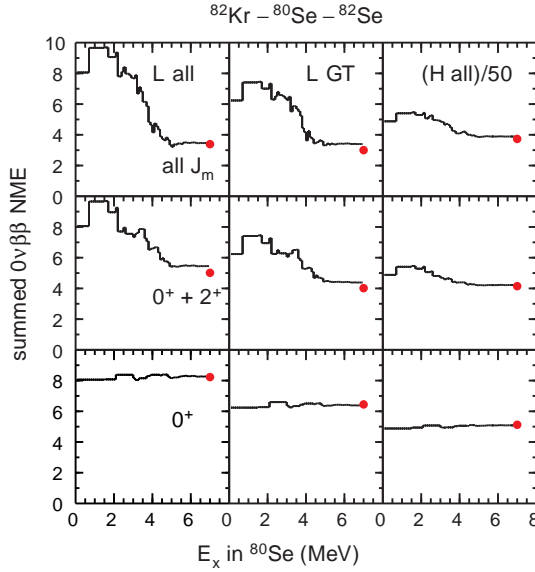


FIG. 5: (Color online) Results for  $^{82}\text{Se}$ . See caption for Fig. 2.

NME is not simple. For example, in the case of  $^{48}\text{Ca}$  the  $J_m = 0^+$  term is dominated by the  $0f_{7/2}$  contribution (top row of Fig. 6). There are small admixtures of the other three orbitals from term (c) in Fig. 6 that change the zero-range direct two-nucleon  $^{48}\text{Ca}$  to  $^{46}\text{Ca}$  transfer amplitude by a factor of 1.48. The (a) and (c) admixtures in Fig. 6 change the  $J_m = 0^+$  NME by a factor of 1.46 (heavy) and 1.24 (light). But when the  $J_m = 2^+$  state is included the NME change by factors of 1.41 (heavy) and 0.89 (light).

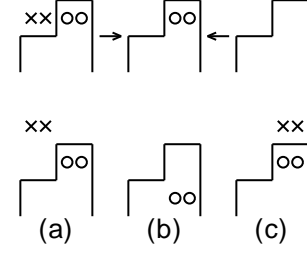


FIG. 6: Schematic diagram for the configuration changes involved in double-beta decay.

There is also the issue of “quenching” relative to the model space. Single-particle transfer [21] and knockout [22] cross sections are usually smaller than those calculated using reaction models with shell-model spectroscopic strengths. This is attributed to short-ranged correlations [23] and particle-vibration coupling [24]. But the connection between quenching for reactions involving single-nucleon overlaps and those involving two-nucleon overlaps is not clear. Experimentally there is some indication from two-particle knockout reactions that there is quenching relative to the shell model for nuclei far from stability [25]. But one should perform knockout experiments for those nuclei involved in double-beta decay to arrive at a consistent picture with the two-particle transfer measurements in the same nuclei [20], [26].

Both the pairing enhancement and quenching issues relative to the model space used in the shell model should be treated consistently in many-body perturbation theory. The first such calculations show an enhancement for the light-neutrino NME of 20% for  $^{76}\text{Ge}$  and 30% for  $^{82}\text{Se}$  [27]. Other models such as QRPA and IBM treat the pairing aspect differently. Perhaps the QRPA NME are larger than the shell-model results since more orbits are included in the pairing. It will be instructive to compare all models used for  $0\nu\beta\beta$  in terms of the size of total NME relative to the  $J_m = 0^+$  contribution from the  $(n-2)$  ground state.

In summary, we have decomposed the neutrinoless double-beta decay matrix elements into sums of products over the intermediate nucleus with two less nucleons. We find that the sum is dominated by the  $J^\pi = 0^+$  ground state of this intermediate nucleus for both the light and heavy neutrino decay processes. We also explain why the light-neutrino NME is dominated by the Gamow-Teller term and show that its TBME are proportional to a simple schematic interactions. This provides new theoretical tools for comparing and improving nuclear structure models and for making connections to two-nucleon transfer and knockout reaction experiments.

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